Mathematica 11.3 Integration Test Results

Test results for the 140 problems in "1.2.4.2 (d x)^m (a x^q+b x^n+c x^(2 n-q))^p.m"

Problem 104: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{\sqrt{a x+b x^3+c x^5}}\,dx$$

Optimal (type 6, 142 leaves, 3 steps):

$$\left(2 x^2 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\ \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b-\sqrt{b^2-4 a c}},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(3 \sqrt{a x+b x^3+c x^5}\right)$$

Result (type 6, 383 leaves):

$$-\left(\left(14 a^2 x^3 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(3 \left(b-\sqrt{b^2-4 a c}\right) \left(b+\sqrt{b^2-4 a c}\right) \left(x \left(a+b x^2+c x^4\right)\right)^{3/2} \left(-7 a\, \text{AppellF1}\left[\frac{3}{4},\frac{1}{2},\frac{1}{2},\frac{7}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{1}{2},\frac{3}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]+\left(b-\sqrt{b^2-4 a c}\right) \text{AppellF1}\left[\frac{7}{4},\frac{3}{2},\frac{1}{2},\frac{11}{4},-\frac{2 c x^2}{b+\sqrt{b^2-4 a c}},\frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)\right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{3/2} \sqrt{a x+b x^3+c x^5}\,dx$$

Optimal (type 4, 380 leaves, 5 steps):

$$-\frac{2\left(b^2-3ac\right)x^{3/2}\left(a+bx^2+cx^4\right)}{15c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{ax+bx^3+cx^5}}+\frac{\sqrt{x}\left(b+3cx^2\right)\sqrt{ax+bx^3+cx^5}}{15c}+$$

$$\left(2a^{1/4}\left(b^2-3ac\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(15c^{7/4}\sqrt{ax+bx^3+cx^5}\right)-$$

$$\left(a^{1/4}\left(2b^2+\sqrt{a}\,b\sqrt{c}-6ac\right)\sqrt{x}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(30c^{7/4}\sqrt{ax+bx^3+cx^5}\right)$$

Result (type 4, 486 leaves) :

$$\frac{1}{30c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{x\left(a+bx^2+cx^4\right)}}\sqrt{x}\left(2c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\left(b+3cx^2\right)\left(a+bx^2+cx^4\right)-\right.$$

$$i\left(b^2-3ac\right)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]+$$

$$i\left(-b^3+4abc+b^2\sqrt{b^2-4ac}-3ac\sqrt{b^2-4ac}\right)$$

$$\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right],\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right]\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{\sqrt{ax+bx^3+cx^5}}{\sqrt{x}}\,dx$$

Optimal (type 4, 347 leaves, 5 steps) :

$$\frac{b\,x^{3/2}\left(a+b\,x^2+c\,x^4\right)}{3\sqrt{c}\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{a\,x+b\,x^3+c\,x^5}}+\frac{1}{3}\sqrt{x}\;\sqrt{a\,x+b\,x^3+c\,x^5}-\left(a^{1/4}\,b\,\sqrt{x}\;\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\;\sqrt{c}}\right)\right]\right)\Big/\left(3\,c^{3/4}\sqrt{a\,x+b\,x^3+c\,x^5}\right)+\left(a^{1/4}\left(b+2\sqrt{a}\;\sqrt{c}\right)\sqrt{x}\;\left(\sqrt{a}+\sqrt{c}\;x^2\right)\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\;x^2\right)^2}}\;\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\;\sqrt{c}}\right)\right]\right)\Big/\left(6\,c^{3/4}\sqrt{a\,x+b\,x^3+c\,x^5}\right)$$

Result (type 4, 452 leaves):

$$\left(\sqrt{x}\left(4\,c\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\left(a+b\,x^2+c\,x^4\right)+\mathrm{i}\,b\left(-b+\sqrt{b^2-4\,a\,c}\right)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-\mathrm{i}\left(-b^2+4\,a\,c+b\sqrt{b^2-4\,a\,c}\right)\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{\frac{2\,b-2\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\;\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\;x\right],\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\right)\Big/\left(12\,c\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\sqrt{x\left(a+b\,x^2+c\,x^4\right)}\right)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{x}\;\left(a\,x+b\,x^3+c\,x^5\right)^{3/2}\,dx$$

Optimal (type 4, 487 leaves, 6 steps):

$$\frac{\left(8 b^4-57 a b^2 c+84 a^2 c^2\right) x^{3/2} \left(a+b x^2+c x^4\right)}{315 c^{5/2} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{a x+b x^3+c x^5}}-$$

$$\frac{\sqrt{x} \left(b \left(4 b^2-9 a c\right)+6 c \left(2 b^2-7 a c\right) x^2\right) \sqrt{a x+b x^3+c x^5}}{315 c^2}+\frac{\left(3 b+7 c x^2\right) \left(a x+b x^3+c x^5\right)^{3/2}}{63 c \sqrt{x}}-$$

$$\left(a^{1/4} \left(8 b^4-57 a b^2 c+84 a^2 c^2\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(315 c^{11/4} \sqrt{a x+b x^3+c x^5}\right)+$$

$$\left(a^{1/4} \left(8 b^4-57 a b^2 c+84 a^2 c^2+4 \sqrt{a} b \sqrt{c} \left(b^2-6 a c\right)\right) \sqrt{x} \left(\sqrt{a}+\sqrt{c} x^2\right) \sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c} x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2 \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2-\frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(630 c^{11/4} \sqrt{a x+b x^3+c x^5}\right)$$

Result (type 4, 609 leaves):

$$\frac{1}{1260 c^3 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{x \left(a+b x^2+c x^4\right)}} \sqrt{x}$$

$$\left(4 c \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x \left(-4 b^4 x^2-b^3 c x^4+53 b^2 c^2 x^6+85 b c^3 x^8+35 c^4 x^{10}+a^2 c \left(24 b+77 c x^2\right)+\right.\right.$$

$$\left.a \left(-4 b^3+27 b^2 c x^2+151 b c^2 x^4+112 c^3 x^6\right)\right)+i \left(8 b^4-57 a b^2 c+84 a^2 c^2\right)$$

$$\left(-b+\sqrt{b^2-4 a c}\right) \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-$$

$$i \left(-8 b^5+65 a b^3 c-132 a^2 b c^2+8 b^4 \sqrt{b^2-4 a c}-57 a b^2 c \sqrt{b^2-4 a c}+84 a^2 c^2 \sqrt{b^2-4 a c}\right)$$

$$\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$

$$\left.\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\left(a\,x+b\,x^3+c\,x^5\right)^{3/2}}{x^{3/2}}\,dx$$

Optimal (type 4, 425 leaves, 6 steps):

$$-\frac{2\,b\,\left(b^2-8\,a\,c\right)\,x^{3/2}\,\left(a+b\,x^2+c\,x^4\right)}{35\,c^{3/2}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{a\,x+b\,x^3+c\,x^5}}+\frac{\sqrt{x}\,\left(b^2+10\,a\,c+3\,b\,c\,x^2\right)\,\sqrt{a\,x+b\,x^3+c\,x^5}}{35\,c}+$$

$$\frac{\left(a\,x+b\,x^3+c\,x^5\right)^{3/2}}{7\,\sqrt{x}}+\left(2\,a^{1/4}\,b\,\left(b^2-8\,a\,c\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(35\,c^{7/4}\,\sqrt{a\,x+b\,x^3+c\,x^5}\right)-$$

$$\left(a^{1/4}\,\left(\sqrt{a}\,\sqrt{c}\,\left(b^2-20\,a\,c\right)+2\,b\,\left(b^2-8\,a\,c\right)\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(70\,c^{7/4}\,\sqrt{a\,x+b\,x^3+c\,x^5}\right)$$

Result (type 4, 540 leaves):

$$\frac{1}{70\,c^2\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{x\,\left(a+b\,x^2+c\,x^4\right)}}\sqrt{x}\,\left(2\,c\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right.$$

$$\left(15\,a^2\,c+a\,\left(b^2+23\,b\,c\,x^2+20\,c^2\,x^4\right)+x^2\,\left(b^3+9\,b^2\,c\,x^2+13\,b\,c^2\,x^4+5\,c^3\,x^6\right)\right)-$$

$$i\,b\,\left(b^2-8\,a\,c\right)\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]+$$

$$i\,\left(-b^4+9\,a\,b^2\,c-20\,a^2\,c^2+b^3\,\sqrt{b^2-4\,a\,c}-8\,a\,b\,c\,\sqrt{b^2-4\,a\,c}\right)$$

$$\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{x}}{\sqrt{a x + b x^3 + c x^5}} \, dx$$

Optimal (type 4, 121 leaves, 2 steps):

$$\left(\sqrt{x} \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(2 \, a^{1/4} c^{1/4} \sqrt{a x + b x^3 + c x^5}\right)$$

Result (type 4, 193 leaves):

$$-\left(\left(i \sqrt{x} \sqrt{\frac{b + \sqrt{b^2 - 4 a c} + 2 c x^2}{b + \sqrt{b^2 - 4 a c}}} \sqrt{1 + \frac{2 c x^2}{b - \sqrt{b^2 - 4 a c}}} \, \text{EllipticF}\left[i \, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \, x\right], \frac{b + \sqrt{b^2 - 4 a c}}{b - \sqrt{b^2 - 4 a c}}\right]\right) \Big/ \left(\sqrt{2} \sqrt{\frac{c}{b + \sqrt{b^2 - 4 a c}}} \sqrt{x \left(a + b x^2 + c x^4\right)}\right)\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{x^{3/2} \sqrt{a x + b x^3 + c x^5}} \, dx$$

Optimal (type 4, 330 leaves, 6 steps):

$$\frac{\sqrt{c} \, x^{3/2} \left(a + b x^2 + c x^4\right)}{a \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{a x + b x^3 + c x^5}} - \frac{\sqrt{a x + b x^3 + c x^5}}{a \, x^{3/2}} - \left(c^{1/4} \sqrt{x} \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} \, x^2\right)^2}} \, \text{EllipticE}\left[2 \, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(a^{3/4} \sqrt{a x + b x^3 + c x^5}\right) + \left(c^{1/4} \sqrt{x} \left(\sqrt{a} + \sqrt{c} \, x^2\right) \sqrt{\frac{a + b x^2 + c x^4}{\left(\sqrt{a} + \sqrt{c} \, x^2\right)^2}} \, \text{EllipticF}\left[2 \, \text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right], \frac{1}{4} \left(2 - \frac{b}{\sqrt{a} \sqrt{c}}\right)\right]\right) \Big/ \left(2 \, a^{3/4} \sqrt{a x + b x^3 + c x^5}\right)$$

Result (type 4, 303 leaves):

$$\left(-4\left(a+b x^2+c x^4\right)+\frac{1}{\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}}\,\mathrm{i}\sqrt{2}\left(-b+\sqrt{b^2-4 a c}\right) x \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}}\sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}}\left(\text{EllipticE}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]-\text{EllipticF}\left[\mathrm{i}\,\text{ArcSinh}\left[\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}}\, x\right],\frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)\right)\Big/\left(4 a \sqrt{x}\sqrt{x\left(a+b x^2+c x^4\right)}\right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{3/2}}{\left(a x+b x^3+c x^5\right)^{3/2}}\, dx$$

Optimal (type 4, 391 leaves, 5 steps):

$$\frac{x^{3/2}\left(b^2-2 a c+b c x^2\right)}{a\left(b^2-4 a c\right)\sqrt{a x+b x^3+c x^5}}-\frac{b\sqrt{c}\, x^{3/2}\left(a+b x^2+c x^4\right)}{a\left(b^2-4 a c\right)\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{a x+b x^3+c x^5}}+\left(b c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(a^{3/4}\left(b^2-4 a c\right)\sqrt{a x+b x^3+c x^5}\right)-\left(c^{1/4}\sqrt{x}\left(\sqrt{a}+\sqrt{c}\, x^2\right)\sqrt{\frac{a+b x^2+c x^4}{\left(\sqrt{a}+\sqrt{c}\, x^2\right)^2}}\,\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4} x}{a^{1/4}}\right],\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right]\right)\Big/\left(2 a^{3/4}\left(b-2\sqrt{a}\sqrt{c}\right)\sqrt{a x+b x^3+c x^5}\right)$$

Result (type 4, 463 leaves):

$$-\frac{1}{4\,a\,\left(b^2-4\,a\,c\right)\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{x\,\left(a+b\,x^2+c\,x^4\right)}}\sqrt{x}\,\left(-4\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\,\left(b^2-2\,a\,c+b\,c\,x^2\right)+\right.$$

$$i\,b\,\left(-b+\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]-$$

$$i\,\left(-b^2+4\,a\,c+b\,\sqrt{b^2-4\,a\,c}\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}\,\sqrt{\frac{2\,b-2\,\sqrt{b^2-4\,a\,c}+4\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}$$

$$\left.\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{2}\,\sqrt{\frac{c}{b+\sqrt{b^2-4\,a\,c}}}\,x\right],\,\frac{b+\sqrt{b^2-4\,a\,c}}{b-\sqrt{b^2-4\,a\,c}}\right]\right)$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{1}{\sqrt{x}\,\left(a\,x+b\,x^3+c\,x^5\right)^{3/2}}\,dx$$

Optimal (type 4, 468 leaves, 6 steps):

$$\frac{b^2-2\,a\,c+b\,c\,x^2}{a\,\left(b^2-4\,a\,c\right)\,\sqrt{x}\,\sqrt{a\,x+b\,x^3+c\,x^5}}+\frac{2\,\sqrt{c}\,\left(b^2-3\,a\,c\right)\,x^{3/2}\,\left(a+b\,x^2+c\,x^4\right)}{a^2\,\left(b^2-4\,a\,c\right)\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{a\,x+b\,x^3+c\,x^5}}-$$

$$\frac{2\,\left(b^2-3\,a\,c\right)\,\sqrt{a\,x+b\,x^3+c\,x^5}}{a^2\,\left(b^2-4\,a\,c\right)\,x^{3/2}}-\left(2\,c^{1/4}\,\left(b^2-3\,a\,c\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticE}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(a^{7/4}\,\left(b^2-4\,a\,c\right)\,\sqrt{a\,x+b\,x^3+c\,x^5}\right)+$$

$$\left(c^{1/4}\,\left(2\,b^2+\sqrt{a}\,b\,\sqrt{c}-6\,a\,c\right)\,\sqrt{x}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)\,\sqrt{\frac{a+b\,x^2+c\,x^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\right.$$

$$\left.\text{EllipticF}\left[2\,\text{ArcTan}\left[\frac{c^{1/4}\,x}{a^{1/4}}\right],\,\frac{1}{4}\,\left(2-\frac{b}{\sqrt{a}\,\sqrt{c}}\right)\right]\right)\Big/\left(2\,a^{7/4}\,\left(b^2-4\,a\,c\right)\,\sqrt{a\,x+b\,x^3+c\,x^5}\right)$$

Result (type 4, 519 leaves):

$$-\frac{1}{2 a^2 \left(b^2-4 a c\right) \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \sqrt{x} \sqrt{x \left(a+b x^2+c x^4\right)}}$$
$$\left(2 \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} \left(-4 a^2 c+2 b^2 x^2 \left(b+c x^2\right)+a \left(b^2-7 b c x^2-6 c^2 x^4\right)\right)-\right.$$
$$\text{i} \left(b^2-3 a c\right) \left(-b+\sqrt{b^2-4 a c}\right) x \sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$
$$\text{EllipticE}\left[\text{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]+$$
$$\text{i} \left(-b^3+4 a b c+b^2 \sqrt{b^2-4 a c}-3 a c \sqrt{b^2-4 a c}\right) x$$
$$\sqrt{\frac{b+\sqrt{b^2-4 a c}+2 c x^2}{b+\sqrt{b^2-4 a c}}} \sqrt{\frac{2 b-2 \sqrt{b^2-4 a c}+4 c x^2}{b-\sqrt{b^2-4 a c}}}$$
$$\left.\text{EllipticF}\left[\text{i}\, \text{ArcSinh}\left[\sqrt{2} \sqrt{\frac{c}{b+\sqrt{b^2-4 a c}}} x\right], \frac{b+\sqrt{b^2-4 a c}}{b-\sqrt{b^2-4 a c}}\right]\right)$$

Problem 122: Result more than twice size of optimal antiderivative.

$$\int \frac{x \left(d+e x^2\right)}{\sqrt{a x+b x^3+c x^5}} \, dx$$

Optimal (type 6, 287 leaves, 7 steps):

$$\left(2 d x^2 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\right.$$
$$\left.\text{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]\right) \Big/$$
$$\left(3 \sqrt{a x+b x^3+c x^5}\right)+\left(2 e x^4 \sqrt{1+\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}} \sqrt{1+\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}}\right.$$
$$\left.\text{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b-\sqrt{b^2-4 a c}}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \left(7 \sqrt{a x+b x^3+c x^5}\right)$$

Result (type 6, 639 leaves):

$$\begin{aligned}
&\frac{1}{42 c \left(x \left(a+b x^2+c x^4\right)\right)^{3/2}} a x^3 \left(b-\sqrt{b^2-4 a c}+2 c x^2\right) \left(b+\sqrt{b^2-4 a c}+2 c x^2\right) \\
&\left(-\left(\left(49 d \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \right.\right. \\
&\quad \left(-7 a \operatorname{AppellF1}\left[\frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{7}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
&\quad\quad x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{3}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
&\quad\quad\quad \left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{7}{4}, \frac{3}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right) - \\
&\quad \left(33 e x^2 \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right) \Big/ \\
&\quad \left(-11 a \operatorname{AppellF1}\left[\frac{7}{4}, \frac{1}{2}, \frac{1}{2}, \frac{11}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
&\quad\quad x^2 \left(\left(b+\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{1}{2}, \frac{3}{2}, \frac{15}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right] + \right. \\
&\quad\quad\quad \left.\left.\left.\left(b-\sqrt{b^2-4 a c}\right) \operatorname{AppellF1}\left[\frac{11}{4}, \frac{3}{2}, \frac{1}{2}, \frac{15}{4}, -\frac{2 c x^2}{b+\sqrt{b^2-4 a c}}, \frac{2 c x^2}{-b+\sqrt{b^2-4 a c}}\right]\right)\right)\right)
\end{aligned}$$

Problem 140: Unable to integrate problem.

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{b x^n + c x^{2 n-q} + a x^q}} \, dx$$

Optimal (type 3, 70 leaves, 2 steps):

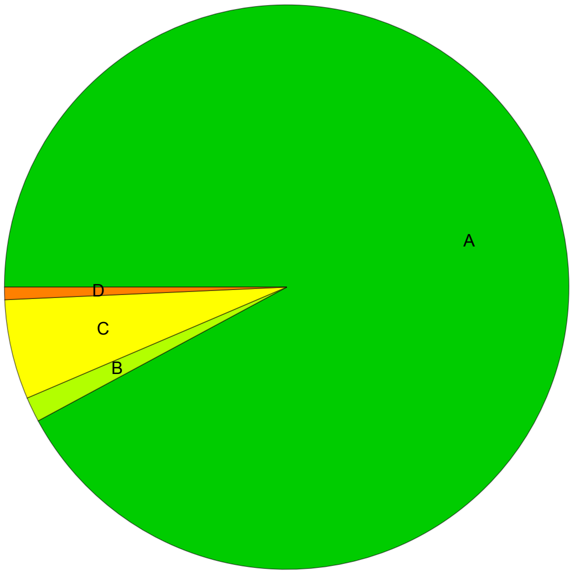
$$-\frac{\operatorname{ArcTanh}\left[\frac{x^{q/2} \left(2 a+b x^{n-q}\right)}{2 \sqrt{a} \sqrt{b x^n+c x^{2 n-q}+a x^q}}\right]}{\sqrt{a} (n-q)}$$

Result (type 8, 38 leaves):

$$\int \frac{x^{-1+\frac{q}{2}}}{\sqrt{b x^n + c x^{2 n-q} + a x^q}} \, dx$$

Summary of Integration Test Results

140 integration problems



A - 129 optimal antiderivatives

B - 2 more than twice size of optimal antiderivatives

C - 8 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts